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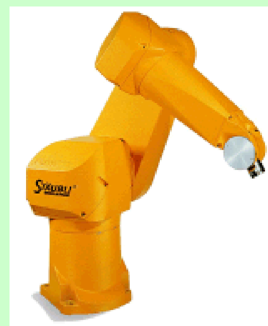
Planeación de trayectorias para optimizar y controlar movimientos

I Los sistemas del Robot
II Cinemática del Robot
  Métodos para el control de movimiento
  Marcos de Referencia
  Cinemática
  Cinemática directa
  Parámetros de Denavit-Hartenberg



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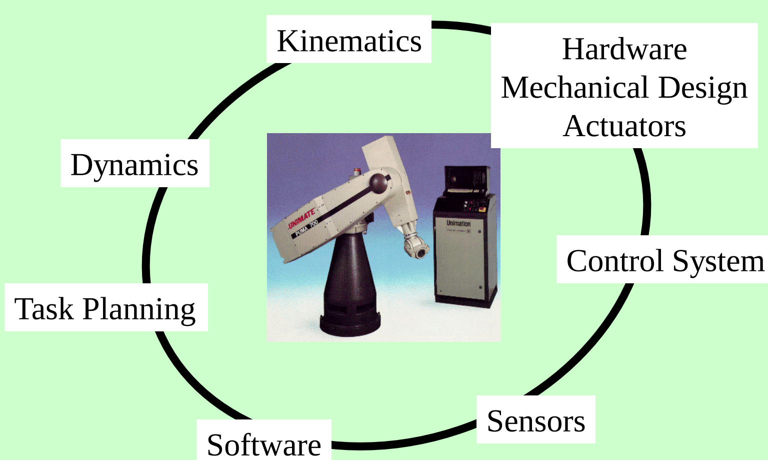
The Robot System



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Robot Kinematics

- In order to control and programme a robot we must have knowledge of both it's ***spatial arrangement*** and ***a means of reference*** to the environment.

- KINEMATICS - the analytical study of the geometry of motion of a robot arm:
  - with respect to a fixed reference co-ordinate system
  - without regard to the forces or moments that cause the motion.

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Motion Control Methods

Point to point control

- a sequence of discrete points
- spot welding, pick-and-place, loading & unloading

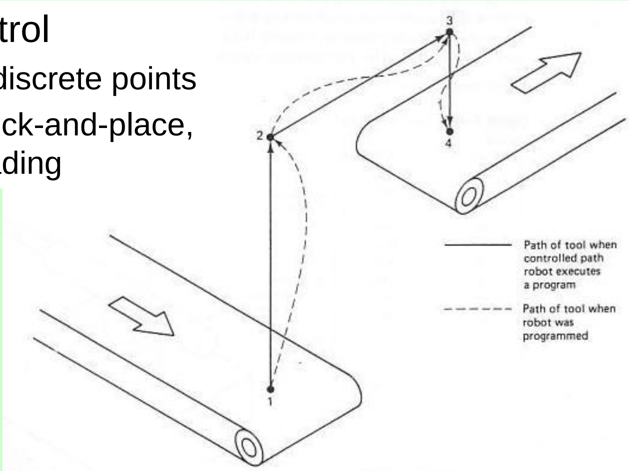

Figure 2–50 Point-to-point Programming Example.

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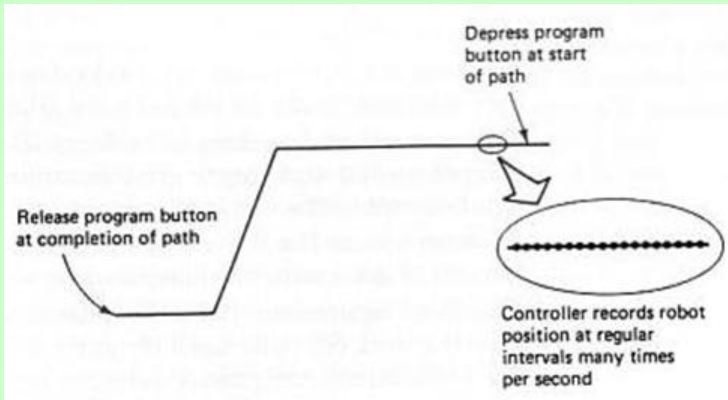
Motion Control Methods

Continuous path control

- follow a prescribed path, controlled-path motion
- Spray painting, Arc welding, Gluing



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Two kinematics topics

Forward Kinematics (angles to position)

What you are given: The length of each link
The angle of each joint

What you can find: The position of any point
(i.e. it's (x, y, z) coordinates)

Inverse Kinematics (position to angles)

What you are given: The length of each link
The position of some point on the robot

What you can find: The angles of each joint needed to obtain that position

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Two kinematics topics

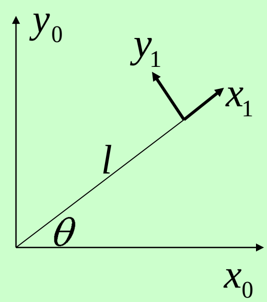
Forward kinematics

$x_0 = l\cos\theta$

$y_0 = l\sin\theta$

Inverse kinematics

$\theta = \cos^{-1}(x_0 / l)$



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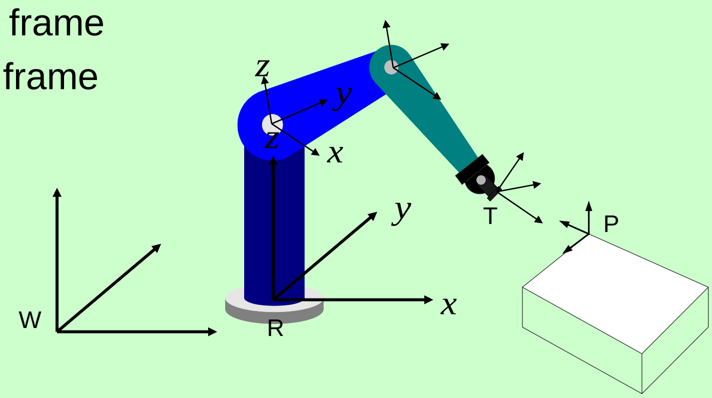
Reference frames

- Robot Reference Frames
  - World frame
  - Joint frame
  - Tool frame



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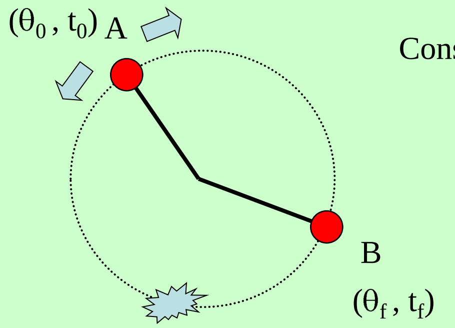
Kinematics

Trajectory Planning



Consider a robot with only one link.

- Kinematics gives one configuration for B.
- Choice of two trajectories to get there.
- May wish to specify a via point - maybe to avoid an obstacle.

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Kinematics

Inverse Kinematics

- For a robot system the inverse kinematic problem is one of the most difficult to solve.
- The robot controller must solve a set of non-linear simultaneous equations.
- The problems can be summarised as:
  - The existence of multiple solutions.
  - The possible non-existence of a solution.
  - Singularities.

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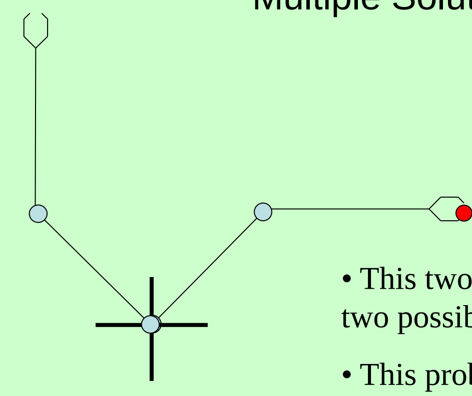
Kinematics

Multiple Solutions



- This two link planar manipulator has two possible solutions.
- This problem gets worse with more ‘Degrees of Freedom’.
- Redundancy of movement.

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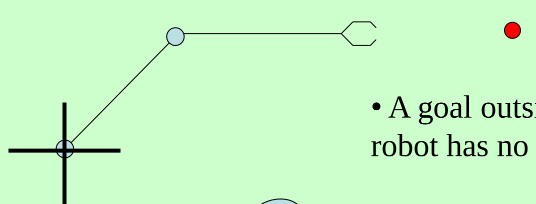
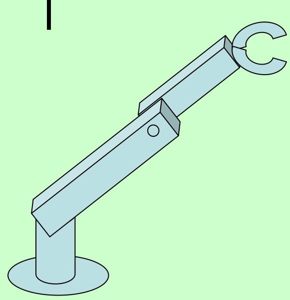
Kinematics

Non Existence of Solution



- A goal outside the workspace of the robot has no solution.



- An unreachable point can also be within the workspace of the manipulator - physical constraints.
- A singularity is a place of ∞ acceleration - trajectory tracking.

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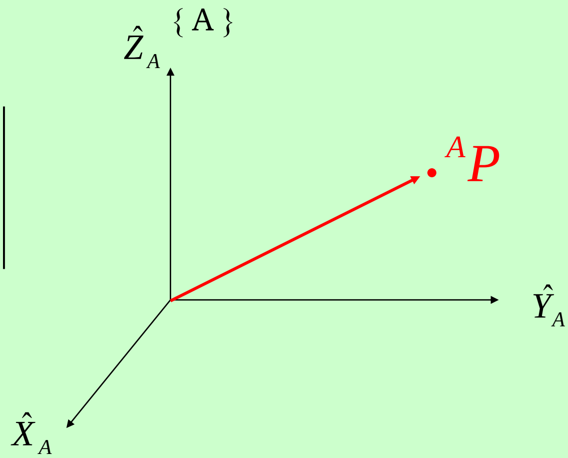
Kinematic

Position:

$$ {}^{A}P = \begin{vmatrix} p_x \\ p_y \\ p_z \end{vmatrix} $$



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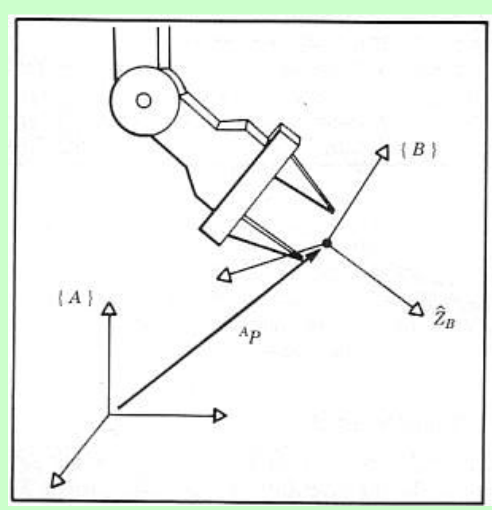
Kinematic

Orientation:

Rotation Matrix describes {B} relative to {A}

$$ {}^{A}_{B}R $$



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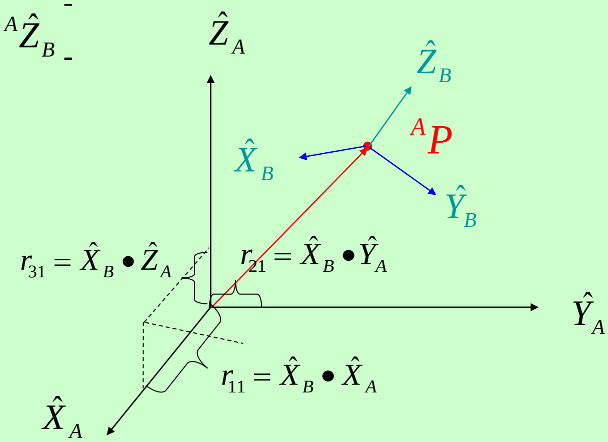
Kinematic

$$ {}^A_B R = \begin{bmatrix} {}^A\hat{X}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \end{bmatrix} $$

$$ = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} $$

Application of the Dot Product



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Kinematic

$$ {}^A_B R = \begin{bmatrix} {}^A\hat{X}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \end{bmatrix} $$

$$ = \begin{bmatrix} \hat{X}_B \bullet \hat{X}_A & \hat{Y}_B \bullet \hat{X}_A & \hat{Z}_B \bullet \hat{X}_A \\ \hat{X}_B \bullet \hat{Y}_A & \hat{Y}_B \bullet \hat{Y}_A & \hat{Z}_B \bullet \hat{Y}_A \\ \hat{X}_B \bullet \hat{Z}_A & \hat{Y}_B \bullet \hat{Z}_A & \hat{Z}_B \bullet \hat{Z}_A \end{bmatrix} $$

Directional Cosines

${}^B\hat{X}_A^T$

Unit vectors of {A} and relative to {B}

Directional Cosines

${}^A\hat{X}_B$

Unit vectors of {B} and relative to {A}

$$ = \begin{bmatrix} {}^B\hat{X}_A^T \\ {}^B\hat{Y}_A^T \\ {}^B\hat{Z}_A^T \end{bmatrix} = \begin{bmatrix} {}^B\hat{X}_A & {}^B\hat{Y}_A & {}^B\hat{Z}_A \end{bmatrix}^T = {}^B_A R^T $$

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Kinematic

$$ {}^{A}_{B}R\,{}^{B}_{A}R = I_3 \Leftrightarrow {}^{A}_{B}R = {}^{B}_{A}R^{-1} $$

$$ \boxed{{}^{A}_{B}R = {}^{B}_{A}R^{T} = {}^{B}_{A}R^{-1}} $$

- For the matrix M,
  - Si $M^{-1} = M^{T}$, M is orthogonal
- Inverse matrix is equal to its transponse.

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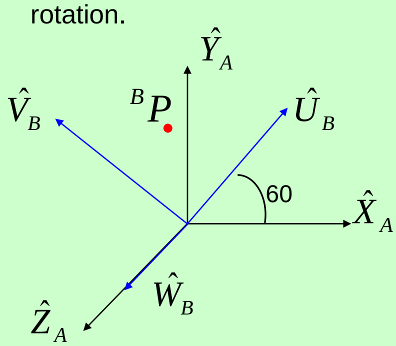
Kinematic

Example:

- A point $a_{uvw} = (4,3,2)$ is attached to a rotating frame, the frame rotates 60 degree about the OZ axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation.



$$ a_{xyz} = {}^{A}_{B}R\,a_{uvw} $$

$$ = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.598 \\ 4.964 \\ 2 \end{bmatrix} $$

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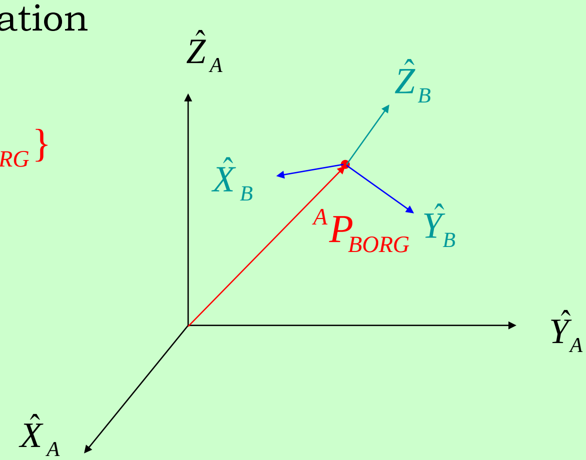
Kinematic

Coordinate system or Frame:

- Position + orientation

$$\{B\} = \{{}^A_B R, {}^A P_{BORG}\}$$



4 vectors

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Kinematic

Mapping from frame to frame:

Only Translation

$${}^A P = {}^A P_{BORG} + {}^B P$$

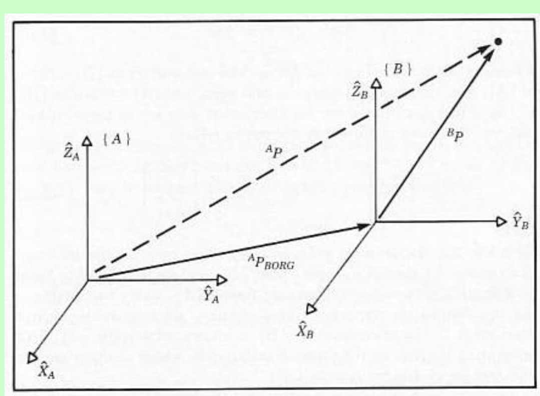

FIGURE 2.4 Translational mapping.

Only when the frame A and B have the same orientation

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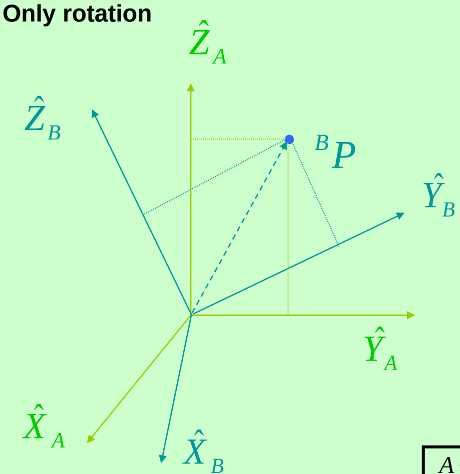
Kinematic

Mapping from frame to frame:

Only rotation



$${}^{B}P = {}^{B}\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = P_x\hat{X}_B + P_y\hat{Y}_B + P_z\hat{Z}_B$$

$$= P_x\,{}^{A}\hat{X}_B + P_y\,{}^{A}\hat{Y}_B + P_z\,{}^{A}\hat{Z}_B$$

$${}^{A}P = {}^{A}\hat{X}_B P_x + {}^{A}\hat{Y}_B P_y + {}^{A}\hat{Z}_B P_z$$

$$= [\,{}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B\,]\;{}^{B}\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

$$= {}^{A}_{B}R\,{}^{B}P$$

$$\boxed{{}^{A}P = {}^{A}_{B}R\,{}^{B}P}$$

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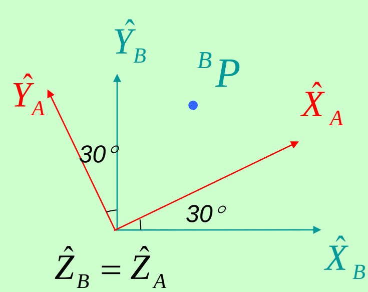
Kinematic

Example:

$${}^{B}P = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow {}^{A}P\,?$$



$${}^{A}_{B}R = \left[\,{}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B\,\right]$$

$$= \begin{bmatrix} \hat{X}_B \bullet \hat{X}_A & \hat{Y}_B \bullet \hat{X}_A & \hat{Z}_B \bullet \hat{X}_A \\ \hat{X}_B \bullet \hat{Y}_A & \hat{Y}_B \bullet \hat{Y}_A & \hat{Z}_B \bullet \hat{Y}_A \\ \hat{X}_B \bullet \hat{Z}_A & \hat{Y}_B \bullet \hat{Z}_A & \hat{Z}_B \bullet \hat{Z}_A \end{bmatrix} = \begin{bmatrix} \cos(-30^\circ) & \cos(60^\circ) & \cos 90^\circ \\ \cos(-120^\circ) & \cos(-30^\circ) & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0^\circ \end{bmatrix}$$

Is it important the angle sign?

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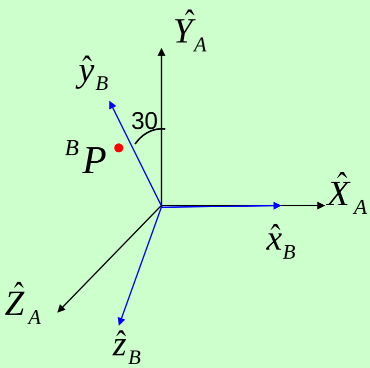
Kinematic

Example:

- A point is at ${}^{B}P_{xyz} = (1,2,1)$ in a body coordinate *B(Oxyz)*. Find the final global position of P after a rotation of *30 deg* about the X-axis of the global coordinate *A(OXYZ)*.



$${}^{A}P = {}^{A}_{B}R\,{}^{B}P$$

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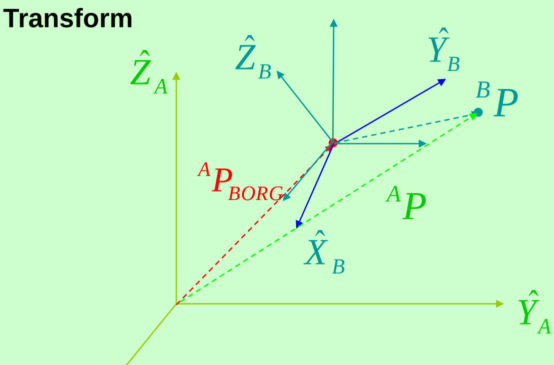
Mapping from frame to frame:

Rotation + translation = Transform



$${}^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

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Kinematic

Rotation + translation

translation → $^{A}P_{BORG}$; *rotation* → $^{A}_{B}R\,^{B}P$

$$^{A}P = {}^{A}P_{BORG} + {}^{A}_{B}R\,{}^{B}P$$

$$^{A}P = {}^{A}_{B}T\,{}^{B}P$$

$$\begin{bmatrix} ^{A}P_x \\ ^{A}P_y \\ ^{A}P_z \\ 1 \end{bmatrix} = \underbrace{\left[\begin{array}{ccc|c} & & & ^{A}P_{BORG\_x} \\ & ^{A}_{B}R & & ^{A}P_{BORG\_y} \\ & & & ^{A}P_{BORG\_z} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]}_{^{A}_{B}T} \begin{bmatrix} ^{B}P_x \\ ^{B}P_y \\ ^{B}P_z \\ 1 \end{bmatrix}$$

(4x4) matrix is called homogenous transform

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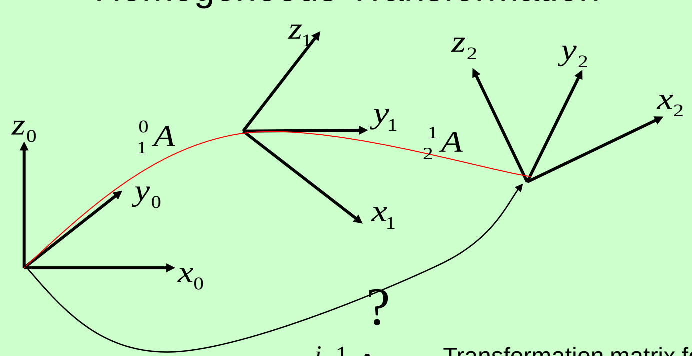
Homogeneous Transformation



$^{i-1}_{i}A$ Transformation matrix for adjacent coordinate frames

$^{0}_{2}A = {}^{0}_{1}A\,{}^{1}_{2}A$ Chain product of successive coordinate transformation matrices

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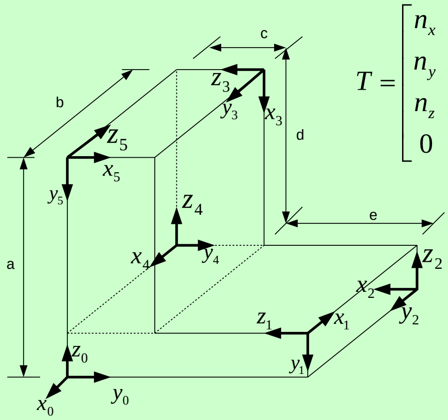
- For the figure shown below, find the 4x4 homogeneous transformation matrices ${}^{i-1}_{i}A$ and ${}^{0}_{i}A$ for i=1, 2, 3, 4, 5



$$T = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{1}A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & e+c \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}_{2}A = \begin{bmatrix} 0 & -1 & 0 & b \\ 0 & 0 & -1 & a-d \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{2}A = \begin{bmatrix} 0 & 1 & 0 & -b \\ -1 & 0 & 0 & e+c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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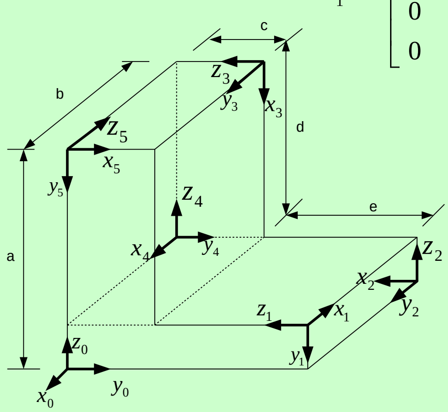
- Results:

$${}^{0}_{1}A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & e+c \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{2}A = \begin{bmatrix} 0 & 1 & 0 & -b \\ -1 & 0 & 0 & e+c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$${}^{0}_{3}A = \begin{bmatrix} 0 & 1 & 0 & -b \\ 0 & 0 & -1 & c \\ -1 & 0 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{5}A = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Orientation Representation

$$T = \begin{bmatrix} R_{3\times3} & P_{3\times1} \\ 0 & 1 \end{bmatrix}$$

- Rotation matrix representation needs 9 elements to completely describe the orientation of a rotating rigid body.
- Any easy way?

Euler Angles Representation

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- Euler Angles Representation (ϕ, θ, φ)
  - Many different types
  - Description of Euler angle representations

	Euler Angle I	Euler Angle II	Roll-Pitch-Yaw
Sequence	ϕ about OZ axis	ϕ about OZ axis	ϕ about OX axis
of	θ about OU axis	θ about OV axis	θ about OY axis
Rotations	φ about OW axis	φ about OW axis	φ about OZ axis

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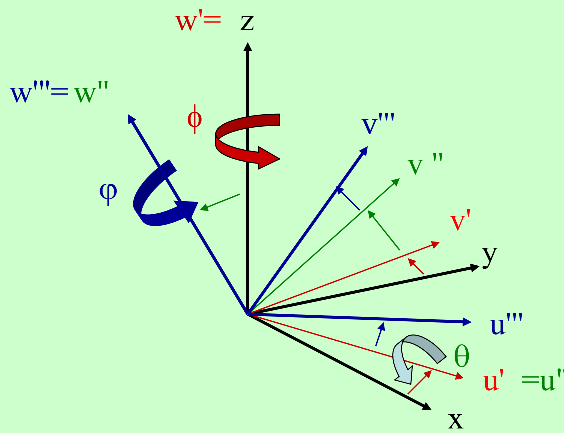
Euler Angle I, Animated



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Orientation Representation

- Euler Angle I

$$R_{z\phi} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_{u'\theta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix},$$

$$R_{w''\varphi} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Euler angle I

Resultant eulerian rotation matrix:

$$R = R_{z\phi} R_{u'\theta} R_{w''\varphi}$$

$$\begin{pmatrix} \cos\phi\cos\varphi - \sin\phi\sin\varphi\cos\theta & -\cos\phi\sin\varphi - \sin\phi\cos\varphi\cos\theta & \sin\varphi\sin\theta \\ \sin\phi\cos\varphi + \cos\phi\sin\varphi\cos\theta & -\sin\phi\sin\varphi + \cos\phi\cos\varphi\cos\theta & -\cos\phi\sin\theta \\ \sin\varphi\sin\theta & \cos\varphi\sin\theta & \cos\theta \end{pmatrix}$$

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Kinematic

Orientation Representation

- Roll-Pitch-Yaw (*fixed angles*)

$$R_{Z\varphi} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_{Y\theta} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix},$$

$$R_{X\phi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}$$

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Roll-Pitch-Yaw Angles

$$ {}^{A}_{B}R_{X,Y,Z}(\phi,\theta,\varphi) = R_Z(\varphi)R_Y(\theta)R_X(\phi) $$

- Matrix for fixed angles:

$$
\begin{bmatrix}
\cos\varphi\cos\theta & \cos\varphi\sin\theta\sin\phi - \sin\varphi\cos\phi & \cos\varphi\sin\theta\cos\phi + \sin\varphi\sin\phi \\
\sin\varphi\cos\theta & \sin\varphi\sin\theta\sin\phi + \cos\varphi\cos\phi & \sin\varphi\sin\theta\cos\phi - \cos\varphi\sin\phi \\
-\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi
\end{bmatrix}
$$

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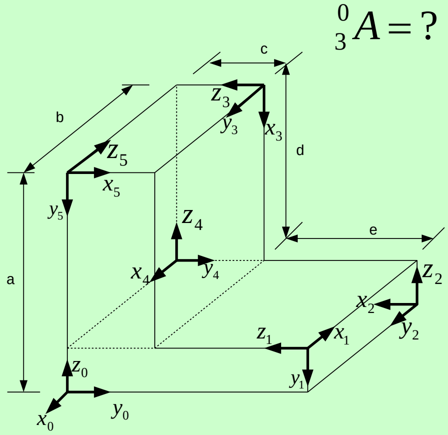
Kinematic

- For the figure shown below, find the 4x4 homogeneous transformation matrix for :

$$ {}^{0}_{3}A = ? $$



$$
{}^{0}_{3}A = \begin{bmatrix}
0 & 1 & 0 & -b \\
0 & 0 & -1 & c \\
-1 & 0 & 0 & a \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

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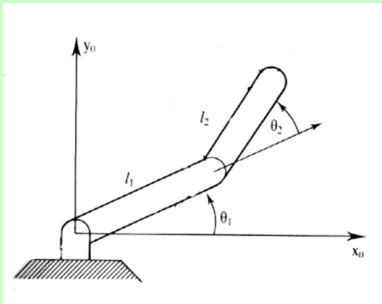
Forward Kinematic



The Situation:
You have a robotic arm that starts out aligned with the x_o-axis.
You tell the first link to move by θ_1 and the second link to move by θ_2.

The Quest:
What is the position of the end of the robotic arm?

Solution:

1. Geometric Approach
This might be the easiest solution for the simple situation. However, notice that the angles are measured relative to the direction of the previous link. For robots with more links and whose arm extends into 3 dimensions the geometry gets much more tedious.

2. Algebraic Approach
Involves coordinate transformations.

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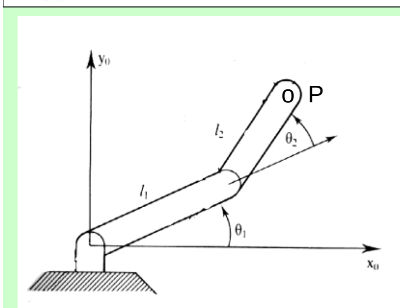
Forward Kinematic



The Situation:
Compute the P position with respect {Xo,Yo,Zo} by considering the Geometric and algebraic approaches. The following information is given:

L_1 = 1 m
L_2 = 0.5 m
θ_1 = 30º
θ_2 = 45º

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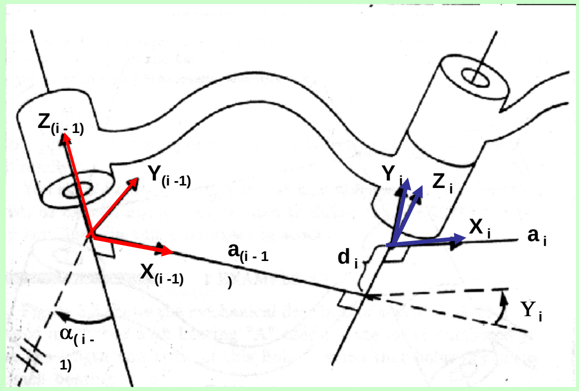
Forward Kinematic

Denavit-Hartenberg Parameters



IDEA: Each joint is assigned a coordinate frame. Using the Denavit-Hartenberg notation, you need 4 parameters to describe how a frame (*i*) relates to a previous frame (*i -1*).

THE PARAMETERS/VARIABLES: **α, *a* , *d*, θ**

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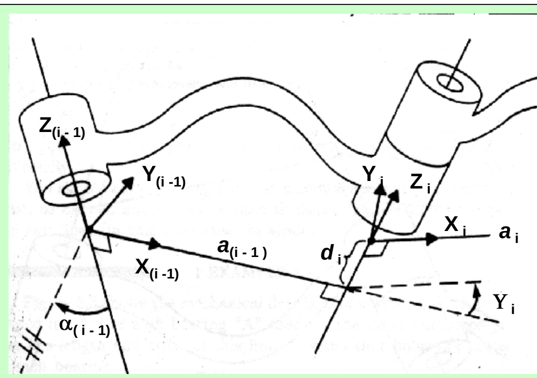
Denavit-Hartenberg Parameters



1) $a_{(i-1)}$

Technical Definition: $a_{(i-1)}$ is the length of the perpendicular between the joint axes. The joint axes are the axes around which revolution takes place which are the $Z_{(i-1)}$ and $Z_{(i)}$ axes. These two axes can be viewed as lines in space. The common perpendicular is the shortest line between the two axis-lines and is perpendicular to both axis-lines.

If the link is prismatic, then $a_{(i-1)}$ is a variable, not a parameter.

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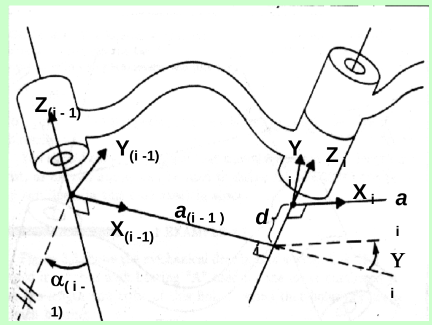
2) $\alpha_{(i-1)}$

Technical Definition: Amount of rotation around the common perpendicular so that the joint axes are parallel (i.e. how much you have to rotate around the $X_{(i-1)}$ axis so that the $Z_{(i-1)}$ is pointing in the same direction as the Z_i axis. Positive rotation follows the right hand rule).

3) d_i

Technical Definition: The displacement along the Z_i axis needed to align the $a_{(i-1)}$ common perpendicular to the a_i common perpendicular. In other words, displacement along the Z_i to align the $X_{(i-1)}$ and X_i axes.



4) θ_i

Amount of rotation around the Z_i axis needed to align the $X_{(i-1)}$ axis with the X_i axis.

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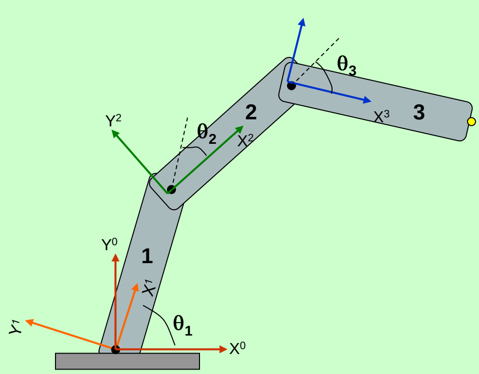
Example Problem:

A three link arm starts out aligned in the x-axis. Each link has lengths l_1, l_2, l_3, respectively. First, link one is rotated by θ_1 , and so on as the diagram suggests. Find the Homogeneous matrix to get the position of the yellow dot with respect to the X^0Y^0 frame.



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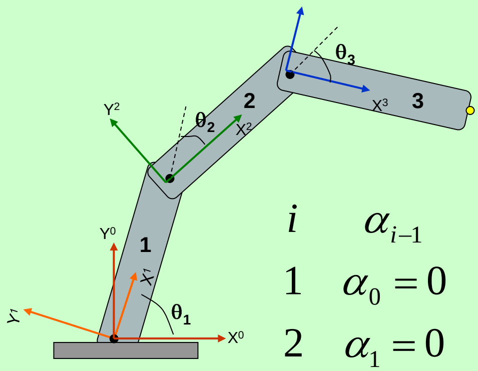

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	$\alpha_0 = 0$	$a_0 = 0$	$d_1 = 0$	$\theta_1 = 30$
2	$\alpha_1 = 0$	$a_1 = L_1$	$d_2 = 0$	$\theta_2 = -15$
3	$\alpha_2 = 0$	$a_2 = L_2$	$d_3 = 0$	$\theta_3 = -40$

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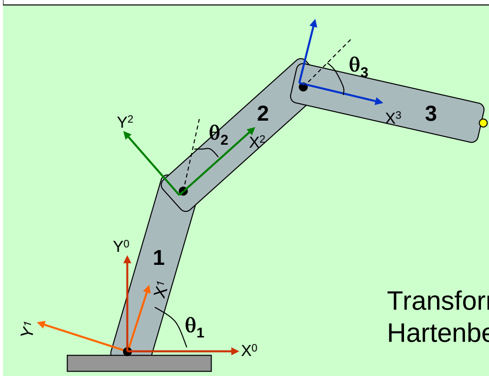

Transformation Matrix with Denavit-Hartenberg parameters

$$
{}^{i-1}_{\;\;i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin(\alpha_{i-1})d_i \\ \sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos(\alpha_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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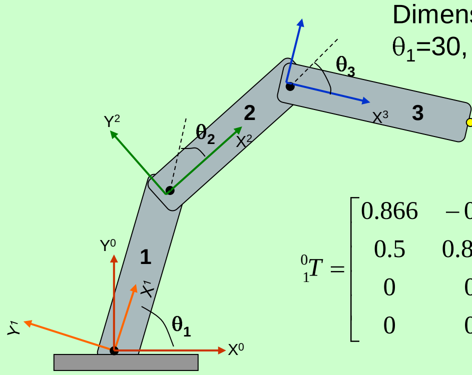

Dimensions: L1=0.8, L2=0.6, θ_1=30, θ_2=-15, θ_3=-40, and

$$^{3}P = \begin{bmatrix} 0.7 \\ 0 \\ 0 \end{bmatrix}$$

Find: $^{0}P = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}P$

$$^{0}_{1}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^{1}_{2}T = \begin{bmatrix} 0.966 & 0.259 & 0 & 0.8 \\ -0.259 & 0.966 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}_{3}T = \begin{bmatrix} 0.766 & 0.643 & 0 & 0.6 \\ -0.643 & 0.766 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^{0}P = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}P = \begin{bmatrix} 1.907 \\ 0.259 \\ 0 \\ 1 \end{bmatrix}$$

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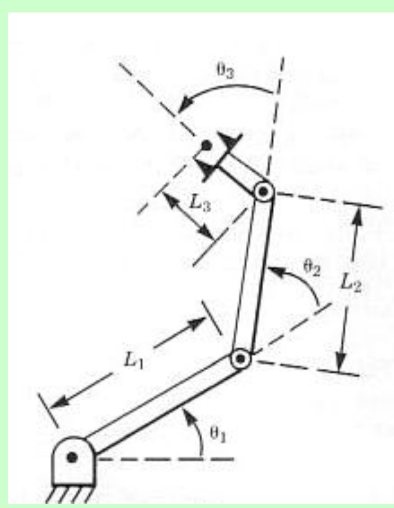
Example 2

Encuentre las transformaciones homogéneas que nos llevarán a encontrar la posición del TCP con respecto al frame de la base.



Dimensions: L1=1.2, L2=0.8, L3=0.5, θ_1=20, θ_2=35, θ_3=45